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Logical Uncertainty: Logical Pluralism and Logical Consequence
A modest proposal for classifying theories of argument

Incertidumbre Lógica: Pluralismo Lógico y Consecuencia Lógica

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Incertidumbre Lógica: Pluralismo Lógico y Consecuencia Lógica

Logical Uncertainty: Logical Pluralism and Logical Consequence

Resumen:

Tradicionalmente se piensa que la lógica no deja lugar a la incertidumbre. La validez de los argumentos y si un enunciado sea una verdad lógica o no, por lo general, no son temas que inviten a tener motivos para dudar. En este artículo argumento que, a pesar de su amplia aceptación, este punto de vista es difícil de mantener. Ofrezco dos razones principales para esta conclusión: (a) A la luz de la pluralidad de lógicas, existen desacuerdos significativos sobre la validez de los argumentos. (b) Es igualmente difícil reconciliar la opinión de que la lógica es cierta con consideraciones en el sentido de que la consecuencia lógica, posiblemente el concepto central de la lógica, no puede analizarse. La naturaleza misma de la consecuencia lógica está, por lo tanto, abierta a dudas. Después de dar algunas ilustraciones en apoyo de (a), discuto un dilema sobre la adecuación de cualquier análisis conceptual de consecuencia lógica, en apoyo de (b), y respondo a algunas posibles objeciones. Al final, la lógica es lo que es independientemente de toda certeza. Cierro con algunas reflexiones sobre por qué esto no es un mal resultado.

Palabras clave: Lógica, incertidumbre, pluralismo lógico, consecuencia lógica.

Abstract:

It is traditionally thought that logic leaves no room for uncertainty. The validity of arguments and whether a statement is a logical truth or not are typically not issues that invite reasons for doubt. In this paper, I argue that, despite its widespread acceptance, this view is difficult to maintain. I offer two main reasons for this conclusion: (a) In light of the plurality of logics, there are significant disagreements about the validity of arguments. (b) It is similarly difficult to reconcile the view that logic is certain with considerations to the effect that logical consequence, arguably the central concept of logic, cannot be analyzed. The very nature of logical consequence is, thus, open for doubt. After giving some illustrations in support of (a), I discuss a dilemma for the adequacy of any conceptual analysis of logical consequence, in support of (b), and respond to some possible objections. In the end, logic is what it is independently of any certainty. I close with some reflections as to why this is not a bad outcome.

Keywords: Logic, Uncertainty, Logical Pluralism, Logical Consequence.

1. Introduction

Logic is traditionally thought of, together with mathematics, as an enterprise where certainty rules. Whether an argument is valid or not, whether a statement is a logical truth or not are issues it is thought don't lead to doubts. No uncertainty should emerge when determining whether a statement follows or not from some premises: either it does or it does not. No uncertainty should emerge when determining whether a statement is a logical truth: either it is or it is not.

Interestingly, with the development of mathematical logic, metatheoretical results about logical systems could be formulated: these gave precision to some limitative results about logic. In particular, first-order logic is not decidable in general, that is, there is no effective procedure to determine whether a statement is a theorem or not of such logic.[1] This is in contrast with propositional logic, which is decidable (the truth-tables provide the relevant procedure). Despite this, the point still stands that for any statement of first-order logic, either it is a theorem of the logic or it is not. There is no uncertainty about that. Whether we *know* that the statement is a theorem, however, is a separate, epistemological matter. That's not a matter regarding logic but of our knowledge of it.

This traditional conception of logic is inadequate. The metalogical results are correct, but the philosophical gloss that is offered based on them is less so. In particular, I argue that considerations based on the plurality of logics and on the difficulty of analyzing the concept of logical consequence, arguably the central concept of logic, offer a very different picture than the one advanced by the traditional conception.[2] As will become clear, analyses of logical

1 George Boolos, John P. Burgess and Richard C. Jeffrey, *Computability and Logic,* 5th ed. (Cambridge: Cambridge University Press, 2007).

2 Otávio Bueno and Melisa Vivanco, "La Lógica y Sus Aplicaciones: ¿Platonismo

consequence either provide a sharp distinction between logical and non-logical notions, or they do not. If a sharp distinction is *not* provided, it is not possible to characterize properly the concept of *logical* consequence, given that, in this case, the concept cannot be sharply distinguished from *mathematical* or *physical* consequence. If a sharp distinction between logical and non-logical notions *is* provided, then the account of consequence will end up presupposing the concept of logical consequence. After all, it is ultimately in terms of the latter that the distinction is drawn. But in this case, the analysis is clearly circular, since it presupposes the very concept that needs to be characterized. As a result, in either case the analysis would be inadequate.

But, I conclude, not everything is lost. The fact that logical uncertainty exists—that is, the fact that there is uncertainty and disagreement about whether arguments are valid and about whether statements are logical truths—does not undermine two central traits of logic: its objectivity and the determination of logical form. In the end, certainty is unnecessary to secure what is really needed from logic.

2. Logical Pluralism

Logical pluralism has been developed, in different versions, for many decades. It only became a genuine philosophical possibility with the emergence of non-classical logics and the realization that specific features of particular contexts may undermine the validity of certain logical inferences.[3] The version of logical

o No-Platonismo?," *Andamios* 16, no. 41 (Septiembre-Diciembre 2019): 19-41; Otávio Bueno, "Is Logic A Priori?," *The Harvard Review of Philosophy* 17, no. 1 (Fall 2010): 105-117; Otávio Bueno, "Revising Logics," in *Logic in Question. Talks from the Annual Sorbonne Logic Workshop (2011- 2019)*, eds. Jean-Yves Béziau et al. (Dordrecht: Birkhäuser, 2022).

3 See Otávio Bueno and Scott A. Shalkowski, "Modalism and Logical Pluralism," *Mind* 118, no. 470 (April 2009): 295-321; Bueno, "Revising Logics."

pluralism I will advance here emerges from Newton da Costa's work, especially his groundbreaking *Essay on the Foundations of Logic*[4] that offers a sophisticated defense of logical pluralism, although one that seems to endorse the idea that each context has its own logic rather than the acknowledgement that pluralism about logic applies even when restricted to a given context. The extended form of pluralism is something that da Costa eventually embraces.[5]

There are a number of reasons for logical pluralism. In what follows, I consider briefly a few of them. (a) The development of non-classical logics is arguably one of the most significant contributions to logic in the 20th century: intuitionistic logic, paraconsistent logic, quantum logic, deontic logic, temporal logic, among so many other logics, have dramatically changed the landscape of logical research. It is a significant fact about logic that a variety of perfectly coherent logical systems can be developed. In this sense, logical pluralism is a fact.

This is no trivial aspect of logic, in contrast with what Priest[6] insists. Clearly, logical pluralism would *not* get off the ground had the plurality of logics been nonexistent. The fact that there are so many logics requires explanation. What is it about logic that allows for the formulation of different logical systems, despite the fact that, for most of its history since Ancient Greece, there has been primarily *one* logic? The proliferation of non-classical logics is clearly a 20th-century phenomenon.

It seems to me that it is the mathematization of logic that allowed for the development of a plurality of logical systems, similarly

4 Newton C. A. da Costa, *Ensaio sobre os Fundamentos da Lógica*, 2nd ed. (São Paulo: Editora Hucitec, 1994).

5 See Newton C. A. da Costa, Otávio Bueno and Steve French, "Is there a Zande Logic?," *History and Philosophy of Logic 19*, no. 1 (1998): 41-54.

6 Graham Priest, *In Contradiction: A Study of the Transconsistent*, 2nd ed. (Oxford: Clarendon Press, 2006).

to what happened in mathematics with the developments of non-Euclidean geometries and various systems of set theory. Once logic is mathematized, and explicit axioms and logical principles are formulated in a system, the question of which of them should or could be revised becomes salient. Note that the revision becomes relevant in light of *motivations* for implementing them: the need to accommodate inconsistencies without triviality (paraconsistent logics), the recognition of objects with incomplete properties (constructive logics), the lack of existential import of the existential quantifier (free logics), violations of distributivity in quantum systems (quantum logics), and so on.

(b) Logical pluralism plays an important role in making sense of certain inferential practices. One cannot make sense of intuitionistic mathematics without constructive logic, nor can inconsistent mathematics be implemented without paraconsistent logic.[7] Certain interpretations of quantum mechanics are unintelligible without non-reflexive logics (logics for which identity is not defined for every object in the domain).[8]

The fact that certain inferential practices require distinct logics calls for a logical pluralist view. A logical monist, who insists that there is only one logic, is unable to make sense of this issue. In fact, the monist does not even *recognize* this to be an issue in the first place.

(c) Logical pluralism allows one to accommodate the scope of various logical principles and rules. Understanding the limits of the scope of logical principles illuminates their strength and the extent to which they apply. We should recognize that there is something odd that from a contradiction everything follows

7 da Costa, *Ensaio sobre os Fundamentos da Lógica*; Newton C. A. da Costa and Otávio Bueno, "Paraconsistency: Towards a Tentative Interpretation," *Theoria* 16, no. 1 (January 2001): 119-145.

8 See Steven French and Décio Krause, *Identity in Physics: A Historical, Philosophical, and Formal Analysis* (Oxford: Oxford University Press, 2006).

(principle of Explosion). Understanding why the principle does not hold in general and why some contradictions may not trivialize a theory illuminates *classical* logic. We understand that it is an artifact of classical logic's consequence relation that it is explosive.

The situation is not significantly different from what happens in the sciences: we understand Newtonian physics better when we realize the limits to its application conditions. With the perihelion of Mercury, relativity theory is able to account for something that cannot be accommodated by Newtonian theory, and one understands why: huge gravitational fields alter the structure of spacetime.

Moreover, the logical consequence relation is arguably the main concept in logical theorizing. Understanding that this concept has significant plasticity and allows for multiple instantiations highlights one of its important traits: its multiple realizability.

An argument is *valid* provided that the conjunction of its premises and the negation of its conclusion is *impossible*. Depending on the *possibility* involved, different logics emerge: (a) If what is possible is what is consistent and complete, classical logic emerges. (b) If what is possible is what is consistent and incomplete, constructive logics result. (c) If what is possible is what is inconsistent and complete, paraconsistent logics emerge. (d) If what is possible is what is inconsistent and incomplete, non-alethic logics result.[9] In this way, despite the plurality of logics, there is a way of systematizing them in modal terms, given what is possible or not in various domains.

9 Bueno and Shalkowski. "Modalism and Logical Pluralism"; Otávio Bueno, "Modality and the Plurality of Logics," in *The Routledge Handbook of Modality*, eds. Otávio Bueno and Scott A. Shalkowski (London: Routledge, 2021), 319-327. For a quantificational account of logical pluralism in terms of cases, see J. C. Beall and Greg Restall, *Logical Pluralism* (Oxford: Clarendon Press, 2006).

Given this plurality, the validity of certain arguments depends on some contexts (or some domains of inquiry, broadly understood). If inconsistent situations are considered, that is, those in which inconsistencies are possible, there are reasons to question the validity of Explosion. Otherwise, in consistent contexts, paraconsistent and classical logics sanction the same inferences as valid. If incomplete situations are considered, that is, those in which incompleteness is possible, there are reasons to question the validity of Excluded Middle. Otherwise, in complete contexts, constructive and classical logics sanction the same inferences as valid.

What this suggests is an interesting form of logical uncertainty. Rather than being applicable indiscriminately to any domain, logics are context-sensitive. Logical principles and inferences hold in some context and fail in others. But the determination of the contexts to which they apply, or fail to apply, is an objective matter, which depends only on the context in question and the relevant logical principles and inferences. As a result, the certainty that has shaped so much of the received view about logic vanishes. Logical objectivity, however, when restricted to particular contexts, still remains.

3. Logical Consequence: A Dilemma

Any conceptual analysis should satisfy two conditions: (i) There is a *pre-theoretical notion* whose analysis we are trying to provide. (ii) The analysis invokes—or is developed in terms of—notions that do *not* presuppose the very notion under consideration. Moreover, the goal of a conceptual analysis is to provide necessary and sufficient conditions to characterize a given concept, and the hope is that the concepts invoked in the analysis are better understood than the concept being analyzed. After all, the program of philosophical analysis involves a genuine search for understanding, and ideally, analyses should be insightful and explanatory.

As an example, consider the analysis of modality in terms of possible worlds: *P* is possible if, and only if, there is a world in which *P*; and *P* is necessary if, and only if, at all worlds *P*.[10] In this case, (i) there is a primitive notion of modality regularly found in ordinary language, and (ii) worlds (at least as conceived of by David Lewis) arguably do not presuppose that notion. Whether Lewis succeeds or not in providing an analysis of modal discourse[11], the project he embarks on clearly provides a systematic approach to the metaphysics of modality.

Whatever the fate of the possible worlds analysis, I argue, in what follows, that the notion of logical consequence cannot be analyzed, indicating along the way why this is the case. For brevity's sake, I will focus, in particular, on *modal* and *model-theoretic* accounts of logical consequence. But the conclusion of the main argument can be easily extended to other accounts as well. In the end, logical consequence is too basic a notion to be analyzed.

The main argument I will advance here explores the contribution played by the distinction (or lack thereof) between logical and non-logical notions to the characterization of logical consequence. The crucial point is that whether such a distinction is assumed or not, the attempt to analyze the notion of logical consequence fails.

In a nutshell, the argument can be expressed as the following dilemma:

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10 See David Lewis, *On the Plurality of Worlds* (Oxford: Blackwell, 1986).

11 See Scott A. Shalkowski, "The Ontological Ground of the Alethic Modality," *The Philosophical Review* 103, no. 4 (October 1994): 669-688; John Divers, *Possible Worlds* (London: Routledge, 2002).

(P1) Either analyses of logical consequence provide a sharp distinction between logical and non-logical notions, or they don't.

(P2) If a sharp distinction is *not* provided, it's not possible to characterize properly the notion of *logical* consequence. (After all, in this case, the latter notion cannot be sharply distinguished from *mathematical*, *metaphysical* or *physical* consequence.)

(P3) If a sharp distinction between logical and non-logical notions *is* provided, the account of consequence ends up *presupposing* the notion of logical consequence, and so it is inadequate. (After all, ultimately it is in terms of the notion of logical consequence that the distinction between logical and non-logical notions is drawn. But in this case, the analysis is clearly circular, since it presupposes the very notion that needs to be analyzed.)

(C) Thus, in either case, the analyses are inadequate.

The argument above is logically valid, so the question is whether we have reason to believe in the truth of the premises. I will consider this issue next.

4. Defending the Premises of the Dilemma

4.1. Defending (P1). Why is (P1) true? The quick response is that (P1) is a logical truth: it's an instance of excluded middle. So, the argument goes, it had better be true.

It might be objected that this response *presupposes* an account of logical truth—and hence of logical consequence—in order to characterize excluded middle *as* a logical truth. And this simply begs the question, given that what is at issue is whether there is an acceptable notion of logical consequence in the first place.

I don't think, however, that the quick response begs the question. The point in question is *not* whether an adequate notion of logical consequence exists. The issue here does *not* concern skepticism about logic. The point is to determine the possibility of *analyzing* the notion of consequence (in the technical sense above). It is accepted that there are logical truths and logical consequences; the trouble is whether we are in a position to *analyze* them. This gives a dialectical advantage to the current proposal over a skeptical view about logic.

However, a further objection could be raised at this point. (P1) invokes a vague term; namely, a *sharp* distinction between logical and non-logical notions. And excluded middle arguably fails in vague contexts. In response, it's important to note that "sharp" is used in a metaphorical sense in (P1). It's not clear that (P1) fails due to the vagueness of "sharp". After all, the crucial issue is whether the *distinction* between logical and non-logical notions is made, rather than whether the distinction is sharp. And whether such a distinction is made or not, (P1) comes out true.

It may be objected that since (P1) is an instance of excluded middle, it is incompatible with a logical pluralist perspective. But this is not the case. After all, a logical pluralist is not a logical nihilist, who denies that anything is logically valid. The pluralist just acknowledges the constraints on the scope of the relevant logical principles, since they fail in particular contexts. However, they work perfectly well elsewhere. And this is precisely what happens with excluded middle in this context, as just indicated.

***4.2. Defending* (*P2*).** Why is (P2) true? Suppose that no distinction is made between logical and non-logical notions. How could we then distinguish *logical* consequence from *mathematical*, *physical* or *metaphysical* consequence?

A vigorous response is provided by the *modal account* of consequence: α is a *logical consequence* of Γ if, and only if, it is not (logically) possible for every member of Γ to be true and α false. We are here talking about *logical* possibility. If we were to consider *mathematical*, *physical* or *metaphysical* possibility, we would obtain the corresponding notions of consequence.

There are two problems with this response, though. First, the response presupposes a *logical* notion of possibility to provide an analysis of the notion of *logical* consequence. But logical possibility is too close to logical consequence to be taken as an adequate starting point for the analysis. After all, to determine whether *P* is *logically possible*, we typically have to establish that no contradiction *logically follows* from *P*. In other words, what is *logically possible* seems to depend on what *logically follows* from what. Thus, it's not clear that the proposed account satisfies condition (ii) of a conceptual analysis, given that it ultimately invokes the notion that the account is trying to analyze.

Second, the modal account will be able to distinguish logical consequence from mathematical, physical or metaphysical consequence *only if there is a distinction between logical and non-logical notions* in the first place. After all, it's in terms of the notions of physical, mathematical or metaphysical possibility (which are arguably non-logical notions) that the modal account distinguishes logical consequence from non-logical consequence. But it was *presupposed* that no such distinction between logical and non-logical notions was made (see (P2), above). In other words, without distinguishing between logical and non-logical notions, the modal account fails to characterize adequately the notion of logical consequence. Therefore, we have (P2).

Here is another way of supporting this point. Consider Gila Sher's model-theoretic characterization of logical constants[12]:

C is a *logical constant* iff *C* is a truth-functional connective or *C* satisfies the following conditions:

(A) A logical constant *C* is syntactically an *n*-place predicate or functor (functional expression) of level 1 or 2, *n* being a positive integer.

(B) A logical constant *C* is defined by a single extensional function and is identified with its extension.

(C) A logical constant *C* is defined over models. In each model **A** over which it is defined, *C* is assigned a construct of elements of **A** corresponding to its syntactic category. Specifically, *C* should be defined by a function f_C such that given a model **A** (with universe *A*) in its domain:

(a) If *C* is a first-level *n*-place predicate, then f_C(**A**) is a subset of A^n.

(b) If *C* is a first-level *n*-place functor, then f_C(**A**) is a function from A^n into *A*.

(c) If *C* is a second-level *n*-place predicate, then f_C(**A**) is a subset of B_1 x … x B_n, where for $n \geq i \geq 1$, $B_i = A$ if *i*(*C*) is an individual, and $B_i = \mathbf{P}(A^m)$ if *i*(*C*) is an *m*-place predicate (*i*(*C*) being the *i*th argument of *C*).

12 Gila Sher, *The Bounds of Logic: A Generalized Viewpoint* (Cambridge, Mass.: MIT Press, 1991), 54-56; "A Characterization of Logical Constants *Is* Possible," *Theoria* 18, no. 2 (May 2003): 189–190.

(d) If C is a second-level n-place functor, then $f_C(\mathbf{A})$ is a function from B_1 x … x B_n into B_{n+1}, where for $n+1 \geq i \geq 1$, B_i is as defined in (c).

(D) A logical constant C is defined over *all* models (for the logic).

(E) A logical constant C is defined by a function f_C which is invariant over isomorphic structures. That is, the following conditions hold:

(a) If C is a first-level n-place predicate, **A** and **A′** are models with universes A and A' respectively, $\langle b_1,\ldots,b_n\rangle \in A^n$, $\langle b'_1,\ldots,b'_n\rangle \in A'^n$, and the structures $\langle A, \langle b_1,\ldots,b_n\rangle\rangle$ and $\langle A',\langle b'_1,\ldots,b'_n\rangle\rangle$ are isomorphic, then $\langle b_1,\ldots,b_n\rangle \in f_C(\mathbf{A})$ iff $\langle b'_1,\ldots,b'_n\rangle \in f_C(\mathbf{A'})$.

(b) If C is a second-level n-place predicate, **A** and **A′** are models with universes A and A' respectively, $\langle D_1,\ldots,D_n\rangle \in B_1$ x ... x B_n, $\langle D'_1,\ldots,D'_n\rangle \in B'_1$ x … x B'_n (where for $n \geq i \geq 1$, B_i and B'_i are as in (C.c)), and the structures $\langle A, \langle D_1,\ldots,D_n\rangle\rangle$ and $\langle A', \langle D'_1,\ldots,D'_n\rangle\rangle$ are isomorphic, then $\langle D_1,\ldots,D_n\rangle \in f_C(\mathbf{A})$ iff $\langle D'_1,\ldots,D'_n\rangle \in f_C(\mathbf{A'})$.

(c) Analogously for functors.

Note that there is a shift between (A) and (B). In (A), a logical constant is an expression, an item of language. In (B), however, we have the constant *identified* with its extension, which typically is not an expression. On Sher's account, expressions such as *finitely many* and *most* turn out to be *logical constants*. Since these are quantifiers that have significant mathematical content, a sharp distinction between logical and mathematical consequence is *not* provided.

***4.3. Defending* (*P3*).** Why is (P3) true? Suppose that a distinction between logical and non-logical notions *is* provided. The *model-theoretic account* of logical consequence yields one way of developing an analysis of logical consequence that presupposes the distinction between logical and non-logical notions. On this account, α is a *logical consequence* of Γ if, and only if, α is true in every model in which every sentence of Γ is true. Ultimately, the truth of the sentences of Γ guarantees the truth of α in virtue of the meanings of the *logical constants* alone.

This approach can be traced back, of course, to Tarski. He emphasized that logical consequence should not depend on empirical knowledge, and cashed this out by insisting on the fact that we could permute all the objects in the domain of interpretation without affecting the consequence relation. As he points out:

> Since we are concerned here with the concept of logical, i.e. *formal*, consequence, and thus with a relation which is to be uniquely determined by the form of the sentences between which it holds, *this relation cannot be influenced in any way by empirical knowledge*, and in particular by *knowledge of the objects to which the sentence* [α] *or the sentences of the class* [Γ] *refer*. The consequence relation *cannot be affected by replacing the designations of the objects referred to in these sentences by the designations of any other objects*.[13] (Emphasis added, except for the italics in 'formal'.)

The description above implicitly assumes an account of the nature of logical notions that Tarski would fully develop some years later. According to this account, logical notions are those that are invariant under all the permutations of the objects in the

13 Alfred Tarski, "On the Concept of Logical Consequence," in *Logic, Semantics, Metamathematics, papers from 1923 to 1938*, trans. J. H. Woodger and ed. John Corcoran, 2nd ed. (Indianapolis: Hackett, 1983), 414-415.

domain of interpretation.[14] And with this account in place, it's then possible to develop fully the model-theoretic characterization of logical consequence.

The problem with the model-theoretic account is that it ultimately *presupposes* that logical notions are not open to reinterpretation. But this means that the distinction between logical and non-logical notions is *assumed* from the start, namely, *logical* notions are not open to reinterpretation; *non-logical* notions are. But why is it that logical notions cannot be reinterpreted? If this was the case, the model-theoretic account wouldn't be extensionally adequate.[15] For example, consider the sentence '$Fa \vee \neg Fa$'. If we were to interpret '$\vee$' by 'whenever', and 'Fa' by 'Graham is in Melbourne', what would *follow* is a sentence that is obviously false. Thus, ultimately it is in terms of the notion of logical consequence that the distinction between logical and non-logical notions is drawn. To avoid an extensionally inadequate account of logical consequence, logical notions have to be taken as fixed. But in this case, the analysis *presupposes* the very notion that needs to be analyzed (the notion of logical consequence), and so it is circular. Once again, the proposal fails to satisfy condition (ii) of a conceptual analysis.

But there is an additional way of reaching the same conclusion. This comes as a response to a criticism of Tarski's account of logical consequence voiced by Vann McGee.[16] According to McGee, it is only with heavy metaphysical assumptions that Tarski's account of logical consequence is extensionally adequate.[17] In McGee's

14 See Alfred Tarski, "What are Logical Notions," *History and Philosophy of Logic* 7, no. 2 (1986): 143-154; Sher, *The Bounds of Logic.*

15 John Etchemendy, *The Concept of Logical Consequence* (Cambridge, Mass.: Harvard University Press, 1990).

16 Vann McGee, "XIII-Two Problems with Tarski's Theory of Consequence," *Proceedings of the Aristotelian Society* 92, no. 1 (June 1992): 273-292.

17 See also Etchemendy, *The Concept of Logical Consequence.*

view, *Tarski's thesis* is the claim that "a sentence of a formalized language is valid just in case it is true in every model (using our current notion of model)".[18] Tarski's thesis can be easily extended to encompass the concept of logical consequence, given that α is valid if, and only if, α is a logical consequence of the empty set. Following McGee and Etchemendy, I will run the discussion in terms of validity (rather than logical consequence). Nothing hangs on that. As McGee points out:

> Intuitively, the statement that φ is logically valid implies that there couldn't be any model in which φ is false. Thus, for Tarski's thesis to be plausible, we shall have to show
>
> > If there might be a model in which φ is false, then there actually exists a model in which φ is false.[19]

To provide an argument for this conclusion, McGee first recalls the theorem to the effect that "for any model, there exists an isomorphic model which is a pure set".[20] He then continues:

> Suppose that there is a possible world *w* in which there is a model **M** in which φ is false. Since the theorem [mentioned above] is true in every world, there exists in *w* a model **L**, isomorphic to **M**, which is a pure set. Being an object of pure mathematics, **L** exists in every world, and it is, in every world, a model in which φ is false. In particular, **L** bears witness to the fact that, in the actual world, there is a model in which φ is false.[21]

Hence, McGee concludes that if there might be a model in which φ is false, then there actually exists a model in which φ is false.

18 McGee, "XIII-Two Problems," 273.

19 McGee, "XIII-Two Problems," 276

20 McGee, "XIII-Two Problems," 276.

21 McGee, "XIII-Two Problems," 276.

Note, first, that McGee *assumes* Tarski's thesis in the above argument. Why is it that the theorem about pure sets "is true in every world"? Because it is valid, and hence—by Tarski's thesis—true in every model. But why is it that what is true in every model is true in every world? That's a version of the problem that McGee's argument is meant to solve. After all, for Tarski's account to go through one would need to establish that: if something is true in every model, then it is true in every world. This result would, of course, allow one to move from truth in every model to truth in every world. But to make this move, *one needs to invoke Tarski's thesis*, which is precisely what allows McGee to justify the claim that the crucial theorem about pure sets is "true in every world". As a result, the overall account becomes circular, given that Tarski's thesis is *assumed* in an argument meant to *support* it—well, at least support it given "heavy metaphysical assumptions".

It might be objected that all that McGee needs in order to justify the claim that the theorem about pure sets holds in every world is to invoke the fact that objects of pure mathematics exist in every world. Hence, given the existence of such objects, the theorem in question will be true in every world. In fact, this is *precisely* part of the metaphysical assumptions made by Tarski's account that McGee highlights.

But this response doesn't quite work. Even if mathematical objects exist in every world, to justify the assertion that the theorem about pure sets is true in every world, one needs to show that in every world the *right sorts of objects* exist. That is, one needs to establish that, *in every world*, for any model, there exists an isomorphic model that is a pure set. And it's not clear how one could establish that without *assuming* that the validity of a theorem entails its truth in every model (via Tarski's thesis), and hence its truth in every world (via McGee's argument). But, in this case, once again, Tarski's thesis has to be assumed.

Now, I'm not claiming that Tarski's account of consequence *doesn't* work. My point is that the account fails to provide a *conceptual analysis* of the notion of logical consequence, given that it *presupposes* the very notion that is being analyzed. The point is beautifully illustrated in McGee's argument—an argument meant to show that Tarski's account of validity only works with strong metaphysical assumptions. In the end, *McGee's argument* only works by presupposing the adequacy of the very notion of validity one is trying to characterize (namely, Tarski's thesis). And exactly the same point applies to the notion of logical consequence.

This illustrates (and supports) the third premise of the dilemma: the fact that the notion of consequence is presupposed in the characterization of this very notion. Interestingly enough, this feature can also be used to illustrate (and support) the dilemma's second premise: without presupposing the notion of logical consequence, no definition of logical consequence can be adequate. Consider again McGee's case. It's only by invoking the very notion of validity that Tarski was trying to analyze (plus some heavy metaphysical assumptions) that McGee was able to establish the adequacy of Tarski's account. Without assuming the notion in question, the proposed analysis doesn't work, given that there's no reason to believe that the relevant models exhaust the logical space. (Of course, given that the analysis *assumes* the notion under consideration, ultimately it fails *as a piece of conceptual analysis*, given that it then becomes circular.)

Could we overcome this problem by avoiding unnecessary metaphysics? Sher's account, which relies on the notion of *formal possibility*, could be invoked for this task. Gil Sagi, however, raised a concern:

> Sher attempts to use formally possible constructions to circumvent possible worlds, or any metaphysical import

> for that matter. Sher's circumvention is only superficial: the most plausible way to understand Sher's use of formally possible constructions is as possible worlds under interpretations of the nonlogical terminology. Thus, the alternative Sher offers to metaphysical and to linguistic semantics is ultimately a combination of both.[22]

If Sagi is right, metaphysical assumptions are still being dragged in through the backdoor once it is specified what formally possible constructions require. Is there an alternative?

I think there is: modalism.[23] Although this is not the place for a defense of the proposal, it is important to highlight a few of its significant features: (a) Modalism (at least in the version I favor) involves very little metaphysics. In particular, no commitment to possible worlds, abstract objects, or universals is to be found—especially when modalism is combined with ontologically neutral quantifiers.[24] (b) A proper account of logical consequence—a modal conception—that reflects the modal character of the logical consequence relation is advanced. (c) No reductive analysis of logical consequence is advanced: primitive modality is assumed throughout (for further details.[25]

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22 Gil Sagi, "Models and Logical Consequence," *Journal of Philosophical Logic* 43, (2014): 957.

23 Otávio Bueno and Scott A. Shalkowski, "Logical Constants: A Modalist Approach," *Noûs* 47, no. 1 (2013): 1-24; Bueno and Shalkowski. "Modalism and Logical Pluralism."

24 Jody Azzouni, *Deflating Existential Consequence: A Case for Nominalism* (New York: Oxford University Press, 2004); Otávio Bueno, "Dirac and the Dispensability of Mathematics," *Studies in History and Philosophy of Science Part B. Studies in History and Philosophy of Modern Physics* 36, no. 3 (September 2005): 465-490.

25 See Bueno, "Modality and the Plurality of Logics."

5. Objections and Replies

To clarify the overall argument put forward above, let me consider some possible objections, and indicate my replies.

5.1. A purely syntactic analysis of consequence? According to a purely syntactic account of consequence, α is a consequence of Γ if, and only if, α is derivable from Γ given certain syntactic rules of derivation. A soundness and completeness theorem then establishes the adequacy of such rules.

Why is it that a purely syntactic analysis of consequence doesn't work? Ultimately, because the notion of derivation presupposes the *adequacy of the rules* invoked in the analysis. Why is it that certain rules are introduced rather than others? Because, one could say, they yield the right consequence relation (or, at least, they yield an extensionally adequate account of that relation). And how do we know what is the right consequence relation? By determining what is provable and what isn't, by the inferences that are made in practice. In the context of pure logic, some indication is given by a soundness and completeness theorem, it might be argued. The trouble here is that the soundness and completeness result for a deductive system only addresses the connection between the syntactic and semantic components of the *formal* language. This result is silent with regard to the relation between the formal language and the pre-theoretical notion of consequence. And *this* is the point where conceptual analysis becomes relevant.[26] But nothing in the syntactic account even begins to address the connection between the pre-theoretical and the formal notions. In fact, it's not clear that the account has the resources to address that connection, given that it simply focuses on the syntactic rules of derivation.

It could be argued that the adequacy of inference rules can only

26 See Etchemendy, *The Concept of Logical Consequence.*

be determined in application conditions, and that such applications are highly context-sensitive and require no pre-theoretical notion of consequence. What matters, in the end, is what goes on in the relevant contexts. In response, it is quite right that to determine the adequacy of a logic in applied contexts, one needs to engage with the details of such contexts. Moreover, in these contexts, logical adequacy ultimately depends on the relevant relations among objects; something that is indeed context-sensitive. It does not follow, nevertheless, that no pre-theoretical notion is involved. If distributivity fails in quantum contexts, it is because of the relations among quantum objects and the quantum-mechanical features they display, e.g., being subject to Heisenberg's uncertainty relations and spin properties.[27] The specification of inferential relations presupposes a notion of consequence, which is pre-theoretical relative to the first formulation of any quantum logic.

Jody Azzouni has vigorously defended a syntactic approach to logical consequence, and challenged the adequacy of a modal account.[28] I think his charges can be resisted, as I'll indicate now.

(a) *Unrestricted syntactic account of logical consequence*. Azzouni argues that, on his favored syntactic approach, there are virtually no restrictions on what counts as a consequence relation, and this includes a tonk operator.[29] This shows that his "account" is clearly not even minimally adequate. After all, there is an intuitive notion of consistency that is used to guide judgments of what follows from what. Not every relation counts as a relation of

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27 da Costa and Bueno, "Paraconsistency: Towards a Tentative Interpretation."

28 See, for instance, Jody Azzouni, *Metaphysical Myths, Mathematical Practice: The Ontology and Epistemology of the Exact Sciences*. (Cambridge: Cambridge University Press, 1994); *Tracking Reason: Proof, Consequence, and Truth* (New York: Oxford University Press, 2006); "Why Deflationary Nominalists and Logical Conventionalists Should Adopt Syntactic Characterizations of Logic and Consequence," accesed May 2022, https://jodyazzouni.com/articles/.

29 Azzouni, "Why Deflationary Nominalists."

logical consequence. The binary relation '*x* is a parent of *y*' clearly is not a consequence relation. A central feature of a consequence relation is that it is not possible to have the conjunction of the premises of a valid argument with the negation of its conclusion, since this amounts to a contradiction. Unless this condition is preserved, what emerges is *not* a relation of logical consequence.

(b) *Model theory, representation of possibilities, and the role of the completeness theorem*. Azzouni[30] also rejects the idea that model theory should be engaged at all in the representation of possibilities. It should secure the correspondence between derivation and semantic consequence via the completeness theorem.

But this is not correct either. Without a proper representation of possibilities, the model-theoretic apparatus would be entirely inadequate to the task at hand, namely, of guaranteeing the impossibility of having the conjunction of the premises of valid arguments and the negation of their conclusion coming out true. If too many possibilities are introduced, relative to those assumed in classical logic, such as allowing for inconsistent or incomplete situations, the resulting account will under-generate, given that explosion or excluded middle are violated, respectively. If too few possibilities are advanced—for instance, if all color predicates are identified as having the same extension in every model—the account will over-generate, given that falsehoods, such as 'Everything is red and green all over', will come out as logical truths. This clearly shows that representation does matter for the model-theoretic account, as it should.

(c) *Modal concepts, unanalyzed notions, and explanatory limitations*. Azzouni[31] also complains that the proposal advanced

30 Azzouni, "Why Deflationary Nominalists."

31 Azzouni, "Why Deflationary Nominalists."

in Bueno and Shalkowski[32] is inadequate, since it involves an unanalyzed notion of possibility. As a result, he claims, it has a significant explanatory limitation.

But this charge also misses the mark. The modalist account defended in Bueno and Shalkowski does have a primitive notion of modality, but this is something that even the syntactic approach needs. After all, the introduction of logical inferences via syntactic rules only advances a piece of pure logic. But it is *applied* logic that needs to be invoked in the assessment of reasoning, and this requires an interpretation of the rules so that they connect properly with their corresponding features in natural language. One may introduce *modus ponens* purely syntactically as: P, $P \rightarrow Q \vdash Q$. Suppose that someone then offers the following as an instance of this rule: "Peter was hanged. If Peter died, then Peter was hanged. Therefore, Peter died". In order to deny that this argument is an adequate instance of *modus ponens*, one needs to insist that the same propositional variables should be assigned to the same natural-language sentences in the same context. Otherwise, clearly invalid arguments, such as the one formulated in natural language just referred to, would count as valid. This means that *possible* substitution instances need to be taken into account in order to make sure that proper use of the syntactic rules is in place. A suitable notion of modality is, thus, presupposed.

Azzouni[33] also claims that Bueno and Shalkowski[34] rely on an assumption about the independence of the model-theoretic formulation of possibilities, regarding what follows from what from, the proof-theoretic formulation of logical truths. But this assumption, for the reasons just discussed, is central to the adequacy of the model-theoretic approach, since it is required

32 Bueno and Shalkowski, "Logical Constants."

33 Azzouni, "Why Deflationary Nominalists."

34 Bueno and Shalkowski, "Logical Constants."

for the proper representation of the relevant possibilities. Absent this feature, the approach doesn't generate the appropriate characterization of logical consequence.

(d) *Our knowledge of Gödel sentences*. Azzouni[35] also complains that the account offered in Bueno[36] of the way in which we have knowledge of the Gödel sentence is not adequate. According to Azzouni, the proposed account relies on the meaning of the Gödel sentence and on an account of intuition that can be problematic for a nominalist. But this is not right. The meaning of the Gödel sentence is not relied on. It is just by understanding what the Gödel sentence states and the diagonal way in which it was constructed—roughly, a formalized version of the statement 'this sentence is not provable'—that we can see that it is true. No problematic notion of meaning is required for that. Moreover, the account of intuition at issue does not depend on the existence of mathematical objects. In fact, it was designed precisely to be acceptable to nominalists.[37]

5.2. Does the dilemma prove too much? It might be argued that the dilemma establishes too much. In fact, the argument goes, the dilemma seems to be perfectly general and, with suitable adjustments, could be used to prove that *no* notion could be analyzed. For either the proposed analysis (whatever it is) is extensionally adequate or it isn't. If it is extensionally adequate, it presupposes what is being analyzed—in terms of which the extensional adequacy is established—and thus, given the resulting circularity, it fails. If the analysis is not extensionally adequate, it

35 Azzouni, "Why Deflationary Nominalists."

36 Otávio Bueno, "Nominalism in the Philosophy of Mathematics," in *The Stanford Encyclopedia of Philosophy*, Stanford University, 1997-, article published September 16, 2013, http://plato.stanford.edu/archives/fall2013/entries/nominalism-mathematics/.

37 For details, see Otávio Bueno, "Nominalism and Mathematical Intuition," *ProtoSociology. An International Journal of Interdisciplinary Research* 25 (2008): 89-107.

also fails, for in this case it is not a proper analysis. Either way, the analysis fails.

There are two responses to this worry. First, I don't see how the dilemma, as formulated earlier in this paper, could be used to establish the impossibility of any conceptual analysis. As we saw, the dilemma depends crucially on the distinction between logical and non-logical notions, and it's not at all clear that that distinction has any bearing on, say, whether the notion of mathematical consequence could be analyzed or not. The distinction between logical and non-logical notions is simply not relevant there. The version of the argument just presented, however, is a formulation of a *different*, more general, argument against the possibility of any conceptual analysis. If the most general argument goes through, then, of course, the analysis of logical consequence would not go through either. But that doesn't mean that the more particular argument discussed earlier in this paper has been generalized. These are distinct arguments.

Second, even if it turned out that the dilemma could be used to establish the impossibility of any conceptual analysis, I wouldn't take this as a *reductio* of the argument. I would actually welcome the result. In this case, the dilemma would highlight an interesting difficulty for a particular philosophical project, the project of philosophical analysis, which although philosophically fruitful, is certainly not without its problems.

Before closing, I'd like to highlight that the primitive notion of modality favored here enters at two crucial points in the discussion of logical consequence. On the one hand, it is found among the various informal arguments in the vernacular, which are the basis for the eventual identification and regimentation in formal inferential patterns. A primitive notion of modality (in this case, of *what follows from what*) is invoked to bring together the various informal arguments. The fact that there is some relative constancy regarding which arguments are intuitively valid and which aren't, despite some disagreement, suggests that a primitive notion does play such a role. Of course, as formal tools are developed, the primitive notion is refined, argument patterns are made explicit, and a justification for which inferences are allowed and which aren't, given a logic, is articulated. Once different logics are formulated, such intuitive notion, with its indeterminacies and imprecisions, is refined further.

On the other hand, a primitive notion of modality is also used to keep in check the adequacy of the various formal frameworks (logics) that are employed to formulate the concept of logical consequence. Central to this task is the fact that each formal framework (each logic) is supposed to represent appropriately the relevant possibilities, so that the resulting views adequately capture the informal arguments that are thought to be intuitively valid. Otherwise, the resulting accounts will characterize as invalid inferences that are intuitively valid, or will characterize as valid inferences that are intuitively invalid. Once again, in light of the development of different logics, the primitive notion that one started with is revisited and polished. It then becomes clear that, depending on the context under consideration, different possibilities are in place and, thus, different logics. Logical pluralism then emerges.[38]

6. Conclusion: Objectivity and Determination of Logical Form without Certainty

For the reasons indicated above, it is not clear that the concept of logical consequence can be analyzed. Although the considerations above focused mainly on *modal* and *model-theoretic* accounts of logical consequence, the main argument—the dilemma—seems

38 See Bueno, "Modality and the Plurality of Logics."

to be general enough to be easily applied to other accounts. Thus, in the end, by highlighting the role played by the distinction between logical and non-logical notions, the above dilemma not only indicates that the notion of logical consequence cannot be analyzed, but also suggests *why* this is the case.

Together with the considerations acknowledging logical pluralism and disagreements about validity and logical truth, the fact that logical consequence cannot be analyzed challenges the unquestioned and alleged certainty of logic. Similarly to what happens in the case of other human endeavors, including mathematics,[39] logic is not immune from doubt. This is not a problem, however, given that two crucial features of logic, its objectivity and (the determination of) logical form, can be obtained independently of certainty.

Certainty is not necessary for objectivity, for logics can be objective even in uncertain contexts. Once a logic is adopted, the issue of whether a conclusion follows or not from some premises does not depend on us. It is a fact about the logic in question and the relevant premises and conclusion. The result is, thus, objective.

Certainty is not necessary for the determination of logical form either. One can determine that the conclusion follows from the premises, since the argument in question exhibits the relevant logical form. This does not prevent disagreement as to whether the relevant logical form is indeed satisfied. Consider, for instance, *modus ponens*. Despite clear cases in which the logical form of particular arguments displays this rule, other cases may be open to interpretation, especially if embedded conditionals are involved. It is not surprising that the arguments used to undermine *modus*

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39 Otávio Bueno, "Contingent Abstract Objects," in *Abstract Objects: For and Against,* eds. José L. Falguera and Concha Martínez-Vidal (Cham: Springer, 2020), 91-109.

ponens involve embedded conditionals[40] and invite disagreement as to whether *modus ponens* is indeed instantiated.[41] This does not change the fact that in contexts in which embedded conditionals are not in place, the logical form of the inference is not an issue at all.

These considerations illustrate that the loss of certainty, if certainty was ever present, is not a significant loss for logics. With objectivity and the determination of logical form still operating, logics can be used without problem even in contexts of uncertainty.

40 See Vann McGee, "A Counterexample to Modus Ponens," *The Journal of Philosophy* 82, no. 9 (September 1985): 462-471; Bueno, "Revising Logics."

41 See E. J. Lowe, "Not a Counterexample to Modus Ponens," *Analysis* 47, no. 1 (January 1987): 44-47.

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