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Supervenience, Entailment, and Vague Objects

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Superveniencia, Implicación y Objetos Vagos

Supervenience, Entailment, and Vague Objects

Resumen: Aunque su uso común sólo ha comenzado recientemente, la relación de superveniencia se está haciendo muy conocida. No obstante, su naturaleza lógica, en particular sus posibles correlaciones con la relación lógica fundamental de implicación, sigue siendo desconocida y necesita clarificación. En este artículo, comparo estas dos relaciones y defino un nuevo acercamiento a la explicación formal de superveniencia. Con esto, utilizo dos concepciones como fuentes principales: sobre la semántica intensional y las descripciones de objetos imposibles como núcleo de aquélla, presentadas en la sección 3; y sobre la relación de consecuencia relevante, descrita brevemente en la sección 4, donde también establezco una nueva interpretación de superveniencia como implicación. Luego, de manera natural, llegamos a descripciones contradictorias e incompletas de objetos, y así, pasa a tomar parte la incertidumbre. De igual manera, permite proponer una definición tentativa de superveniencia sin hacer referencia a la terminología de la semántica de los mundos posibles.

Palabras clave: Superveniencia, descripción de objetos, vinculación, semántica intencional, incertidumbre.

Abstract: Although it's only recently come into common use, the relation of supervenience is rapidly gaining in popularity. At the same time, its logical nature, in particular its possible correlations with such fundamental logical relation as entailment, remains unresolved and needs clarification. In this paper, I compare these two relations and outline a new approach to formal explication of supervenience. In so doing, I employ as main sources two conceptions: of intensional semantics, and impossible object descriptions as its core part, introduced in section 3, and of relevant consequence relation, briefly described in section 4, where I also delineate a new entailment interpretation of supervenience. Thus, quite naturally we arrive at contradictory and incomplete descriptions of objects, and that way, uncertainty comes into play. Equally, it allows to propose a tentative definition of supervenience without referring to the terminology of the possible worlds semantics.

Keywords: Supervenience, object description, entailment, intensional semantics.

1. Motivation and Preliminary Considerations

Supervenience is one among key notions in modern analytic philosophy and philosophy of mind. It is a kind of determination or dependence relation, which was aphoristically characterized by David Lewis's¹ oft-quoted saying: "supervenience means that there could be no difference of one sort without difference of the other sort". Such a sophisticated dependence relation is involved in an impressive number of philosophical issues, when it is needed to clarify the relation of some higher-level properties to lower-level ones. For example, some people think that:

- aesthetic, moral and mental properties supervene upon physical properties;
- future supervenes on the past and natural laws;
- modal truths supervene on non-modal truths;
- general truths supervene on particular truths, etc.

In their zeal to explicate the concept, highly-rated researches proposed different definitions of the term as well as discerned varieties of supervenience. Thus a distinction is made between individual, regional, global, similarity-based, multiple domain versions of supervenience, all in weak and strong forms. For most of its short, but eventful and rich in ideas history, supervenience is associated with the contexts of uncertainty and vagueness. In his seminal paper² B. P. McLaughlin proposed to examine supervenience in the context of uncertainty, more precisely, the idea was to "consider supervenience relations involving vague properties and/or predicates"³. That said, in the next sentence, he comments that this is not some special case of supervenience, but the typical one: in fact, all predicates are vague. Though there are a variety of opinions on the ratio of uncertainty and

1 David K. Lewis, *On the plurality of worlds* (Oxford: Blackwell, 1986), 15.

2 Brian P. McLaughlin, "Supervenience, vagueness, and determination," *Philosophical perspectives* 11 (1997): 209-230.

3 McLaughlin, "Supervenience," 219.

vagueness, for the purpose of this paper, these differences are not so fundamental. Here it may just be noted that it is quite justifiable to interpret vagueness as a kind of uncertainty, when the agent's lack of knowledge is due to the specifics of the concepts (and corresponding objects) under consideration. One way or another, whether an object is inherently vague (as, for example, quantum objects⁴), or our knowledge of it contains gaps and contradictions, ultimately, in an epistemological perspective, we always face an ambiguous description of the object of cognition. A good example of philosophical meditations on non-standard descriptions of objects in the context of finite individuality and objective uncertainty is presented in the recent paper by M. Ryabkov⁵.

I would like to preface further consideration with the following important note. My task here is not to formally explicate different types of supervenience, but rather to propose a new general approach to such explication. That way, in this paper, I choose one of the most common definitions of supervenience and demonstrate the possibilities of the developed approach with this example. Thus, presented below, there is a preliminary version deliberately simplified to highlight the very idea of formal presentation of supervenience relation without possible worlds by means of impossible objects descriptions.

In what follows I will present a new interpretation of supervenience relation (1) by means of relevance entailment (consequence relation), (2) substantially exploiting the ideas of the semantics of object descriptions. In the next second section, I will continue with traditional firm views on interrelation between supervenience and entailment. The third section will be devoted to a short overview of the semantics of object descriptions to the extent that it is appropriate, and on this basis I will introduce the preliminary version of supervenience entailment relation. The final section

4 Steven French and Décio Krause, "Quantum objects are vague objects," *Sorites*, no. 6 (August 1996): 21-33.

5 Maxim Ryabkov, "Paradox of the duplication of physical information," *Humanities & Social Sciences Communications* 8, no. 143 (June 2021): 1-8, <https://doi.org/10.1057/s41599-021-00803-z>.

presents a brief semantical characteristic of relevant entailment, which allows developing relevant interpretation of supervenience.

2. Supervenience and Entailment at first Glance

In the context of current research, I will adhere to the interpretation of supervenience, going back to the well-known formulation of J. Kim⁶:

“a set F of properties is supervenient upon a set G of properties with respect to a domain D just in case any two things in D which are indiscernible with respect to G are necessarily indiscernible with respect to F (that is to say, any two things in D are such that necessarily if they differ with respect to F then they differ with respect to G)”.

Summing up, one can say that G-indiscernibility necessarily implies F-indiscernibility.

Before going ahead, it should be fixed what the term ‘necessarily’ refers to in Kim’s formulation. The explanation proposed by him in parentheses in the above quote, to my way of thinking, clearly indicates that what is meant here is the entailment relation. Literally the linguistic construction ‘necessarily if... then...’ denotes necessary implication. There are different interpretations of the relationship between logical implication as a semantic consequence relation and entailment. The simplest option is to identify them completely, but the position expressed by M. Dunn and G. Restall⁷ seems more farsighted and fruitful: “Thus we tend to differentiate ‘entails’ from ‘implies’ on precisely the ground that ‘entails’, unlike ‘implies’, stands only for

6 Jaegwon Kim, “Psychophysical supervenience,” *Philosophical studies: An International Journal for Philosophy in the Analytic Tradition* 41 (January 1982): 51.

7 J. Michael Dunn and Greg Restall, “Relevance logic,” en *Handbook of philosophical logic*, ed. Dov M. Gabbay and F. Guenther (Dordrecht: Springer, 2002), 5.

necessary implication”. That is, ‘A entails B’ means (1) that under any interpretation of non-logical terms, if A is true then B is true; and (2) this connection is necessary. There is a long tradition in logic to consider the consequence relation as a necessary relation based on the meaning of propositions. For example, for Abelard, “[a]n inferentia holds when the premises (or, in Abelard’s case the antecedent) necessitate the conclusion (consequence) in virtue of their meaning”.⁸ I also adhere to this tradition in the formal interpretation of supervenience being developed, avoiding at this point direct appeal to modal operators.

Keeping in mind that entailment as a necessity consequence relation can be represented via implication (\rightarrow) as its object-language counterpart, formal explication goes as follows:

DEF 1.

A set of properties **B** supervenes upon a set of properties **A** $\Leftrightarrow \forall x \forall y (\forall F (F \in \mathbf{A} \rightarrow (F(x) \equiv F(y))) \rightarrow \forall G (G \in \mathbf{B} \rightarrow (G(x) \equiv G(y))))$

To make this definition easy-to-read (that is, first of all to make it easy-to-see), define indiscernibility with respect to any set of properties **A** ($x \approx_{\mathbf{A}} y$) in Leibnizian tradition.

DEF 2.

$x \approx_{\mathbf{A}} y \Leftrightarrow \forall F (F \in \mathbf{A} \rightarrow (F(x) \equiv F(y)))$

Now the **DEF 1** can be presented in a more readable way.

8 Conrad Asmus and Greg Restall, “A history of the consequence relations,” in *Logic: a history of its central concepts*, vol. 11 of *Handbook of the history of logic*, ed. Dov M. Gabbay, Francis Jeffrey Pelletier and John Woods (North-Holland: Elsevier, 2012), 22. Besides, this chapter contains interesting considerations of the concepts of necessity, analytic and *a priori* in a context of the consequence relation.

DEF 1*.

A set of properties **B** supervenes upon a set of properties **A** \Leftrightarrow
 $\forall x \forall y (x \approx_A y \rightarrow x \approx_B y)$

There are at least two considerations that stimulate a comparison of entailment and supervenience. First of all, the former relation in the form of universal implication is presented in the above definition of the latter. Secondly, both of them express certain kinds of dependence (between premises and conclusion or two sets of properties, respectively), which are interesting to compare. I am far from being the first to consider these two relations. For an exhaustive information on the subject consult subsection of SEP entry ‘Supervenience’, entitled ‘Supervenience and Entailment’ by McLaughlin and Bennett.⁹ The main theses are as follows:

1. both relations are reflexive transitive and non-symmetric;
2. consequence relation does not suffice for supervenience;
3. supervenience does not suffice for consequence relation.

To begin with, it is non-symmetry of entailment (or consequence) relation, what gives rise to doubt. I used to consider entailment (at least in a standard case) as an instance of partial-ordering relation that is reflexive, transitive and *antisymmetric*. In particular, if $A \models B$ and $B \models A$, these formulas are truth-functionally equivalent, and hence for any assignment their truth-values coincide, thus $A = B$. Judging by the text, it is no more than an inaccurate word-usage: ‘non-symmetric relation’ means for the authors that “sometimes it holds symmetrically. ... And

⁹ Brian McLaughlin and Karen Bennett, “Supervenience,” in *The Stanford Encyclopedia of Philosophy*, Stanford University, 1997-, article published July 25, 2005; last modified January 10, 2018, <https://plato.stanford.edu/archives/sum2021/entries/supervenience/>.

sometimes it holds asymmetrically”. One way or another, to this extent both supervenience and entailment instantiate the same abstract binary relation.

The second thesis is illustrated by a well-known example of disjunctive property, namely being a sibling. For any person, being a brother entails being a sibling, but the latter does not supervene on the former. The same holds for conjunctive properties. Let $\mathbf{B} = \{P \wedge Q\}$, $\mathbf{A} = \{P, Q\}$, and the following holds: $P(a), Q(b), \neg P(b), \neg Q(a)$. Then a and b are \mathbf{B} -indiscernible without being \mathbf{A} -indiscernible.

To show that \mathbf{A} can supervene on \mathbf{B} , even though \mathbf{B} -properties do not entail \mathbf{A} -properties, it is instructive to consider negative properties (if one accepts their existence). It is evident that $\mathbf{B} = \{\neg P\}$ supervenes on $\mathbf{A} = \{P\}$, while $\neg P$ does not entail P .

As a result of their consideration the authors of above mentioned SEP entry come to a conclusion that “the logical supervenience of property set \mathbf{A} on property set \mathbf{B} will only guarantee that each \mathbf{A} -property is entailed by some \mathbf{B} -property if \mathbf{A} and \mathbf{B} are closed under both infinitary Boolean operations and property-forming operations involving quantification”. In accordance with this upshot I would like to accept certain commitments concerning the linguistic framework for my approach.

1. Atomic properties are unqualified literals (positive and negative properties).
2. Compound properties are constructed by means of propositional connectives.
3. For a set of properties \mathbf{A} including a finite number of properties C_1, C_2, \dots, C_n , thereafter A stands for a compound conjunctive property $C_1 \wedge C_2 \wedge \dots \wedge C_n$.

3. Semantics of Object Descriptions

The underlying ideas of object descriptions can be found in early book by Vladimir Markin¹⁰. Subsequently, this approach applied to syllogistics was further developed in the more recent papers of him¹¹ and his students¹². In a sense it can be considered as formal realization of Leibnitz' idea of intensional interpretation of the traditional syllogistic. Though, there are some more direct attempts to formalize precisely Leibnizian intuition regarding categorical propositions in form of intensional semantics¹³, in this paper, I will follow genuine interpretation of intensional semantics coined by Markin.

The core idea of Markin's intensional semantics is to associate with each term not its extension being a set of objects but rather its intension – a set of (positive and negative) properties. At this rate, an arbitrary object can be determined by infinite set of properties, where a unique set corresponds to every object. It is this characteristic set of properties, what is labeled as 'object description'.

To introduce formally the idea of object description α , consider a set of literals L .

DEF. 4 Let $W = \{\alpha \mid \alpha \subseteq L\}$, where

$$\forall P_i \in L (P_i \in \alpha \text{ or } \neg P_i \in \alpha)$$

$$\forall P_i \in L (P_i \notin \alpha \text{ or } \neg P_i \notin \alpha).$$

10 Vladimir Markin, *Syllogistic Theories in Modern Logic* [in Russian] (Moscow: MSU, 1991).

11 Vladimir Markin, "What trends in non-classical logic were anticipated by Nikolai Vasiliev?," *Logical Investigations* 19, no. 1 (2013): 122-135.

12 Antonina Konkova and Maria Legeydo, "Intensional Semantics for Syllogistics: what Leibniz and Vasiliev Have in Common," *Logic and Logical Philosophy* (2022). *To appear*.

13 Klaus Glashoff, "An intensional Leibniz semantics for Aristotelian logic," *The Review of Symbolic Logic* 3, no. 2 (June 2010): 262-272. Robert van Rooij, "Leibnizian intensional semantics for syllogistic reasoning," en *Recent Trends in Philosophical Logic*, ed. Roberto Ciuni, Heinrich Wansing and Caroline Willkommen (Cham: Springer, 2014), 179-194.

Conditions (1) and (2) mean that object descriptions are complete and consistent, correspondingly. Hence, these conditions prevent the existence of impossible objects. The formal explication proposed above may be easily extended to compound statements.

For a set of singular terms N , define interpretation function d , $\forall k_n \in N$, $d(k_n) \in W$, and truth (falsity) for arbitrary statement A inductively:

DEF 5.

$$A := P(k)$$

$$|P(k)|_d = \mathbf{t} \Leftrightarrow P \in d(k); \quad |P(k)|_d = \mathbf{f} \Leftrightarrow \neg P \in d(k).$$

$$A := \neg B$$

$$|\neg B|_d = \mathbf{t} \Leftrightarrow |B|_d = \mathbf{f}; \quad |\neg B|_d = \mathbf{f} \Leftrightarrow |B|_d = \mathbf{t}.$$

$$A := B \wedge C$$

$$|B \wedge C|_d = \mathbf{t} \Leftrightarrow |B|_d = \mathbf{t} \text{ and } |C|_d = \mathbf{t};$$

$$|B \wedge C|_d = \mathbf{f} \Leftrightarrow |B|_d = \mathbf{f} \text{ or } |C|_d = \mathbf{f}.$$

Now $\forall A \forall d |A(x)|_d = |A(y)|_d$ may be interpreted as an appropriate form of indiscernibility, because it asserts that two individuals x and y equally possess (or do not possess) all relevant properties.

All the above makes it possible to introduce supervenient (consequence) relation (\vDash_s) in terms of object descriptions semantics.

DEF 6.

$$A \vDash_s B \Leftrightarrow \forall d \forall x \forall y (|A(x)|_d = |A(y)|_d \Rightarrow |B(x)|_d = |B(y)|_d)$$

However such consequence relation is not free from paradoxes, which arise due to classical treatment on implication. If any two

objects equally exhibit or do not exhibit certain property, then the consequent of the definiens is true and, hence, the conditional is also true. For example, consider the following: $\forall A (A \vDash_s B \wedge \neg B)$. Evaluated via complete and consistent object descriptions a compound property $B \wedge \neg B$ is inherent to no object at all. It leads to $|B(x) \wedge \neg B(x)|_d = |B(y) \wedge \neg B(y)|_d = \mathbf{f}$, and hence the consequent of the definiens is true, thus in turn, $A \vDash_s B \wedge \neg B$.

To avoid such paradoxes, one is committed to delete conditions (1) and (2) in the characteristics of object description, and as a result arrive at the idea of generalized object description being a description of impossible (vague) object. Indeed objects introduced by means of incomplete and inconsistent descriptions may be contradictory, as the rejection of the second conditions permits possessing B and *not-B* at the same time, or uncertain in the sense that for some property B and for some object k , neither B nor *not-B* is shared by k .

Such a small refinement had serious consequences – now an arbitrary statement may be only true, only false, both true and false, and neither. These four new values are compound and represent all possible combinations of initial atomic values \mathbf{t} and \mathbf{f} , so DEF 5 needs minimal relevant adjustment:

DEF 5*.

$$A := P(k)$$

$$\mathbf{t} \in |P(k)|_d \Leftrightarrow P \in d(k); \quad \mathbf{f} \in |P(k)|_d \Leftrightarrow \neg P \in d(k).$$

$$A := \neg B$$

$$\mathbf{t} \in |\neg B|_d \Leftrightarrow \mathbf{f} \in |B|_d; \quad \mathbf{f} \in |\neg B|_d \Leftrightarrow \mathbf{t} \in |B|_d.$$

$$A := B \wedge C$$

$$\mathbf{t} \in |B \wedge C|_d \Leftrightarrow \mathbf{t} \in |B|_d \text{ and } \mathbf{t} \in |C|_d;$$

$$\mathbf{f} \in |B \wedge C|_d \Leftrightarrow \mathbf{f} \in |B|_d \text{ or } \mathbf{f} \in |C|_d.$$

Condition for disjunction is a trivial combination of conditions for negation and conjunction, where $B \vee C \equiv \neg(\neg B \wedge \neg C)$. Summing up, at the moment we have a paradoxes-free definition of supervenience in a form of consequence relation. However, to the best of my knowledge, there is no logical system axiomatizing this consequence relation. At the same time, the idea of contradictory and incomplete assignments evokes famous Dunn and Belnap's useful four-valued logic¹⁴ with equally famous axiomatizations known as E_{FDE} and R_{FDE} (where FDE stands for First Degree Entailment) being two deductively equivalent formulations of the first-degree fragment of all relevant logics. To clarify the properties of so defined consequence relation, we proceed further and, first, briefly introduce the idea of generalized state descriptions and based on it semantics of FDE, secondly, express supervenience relation in terms of relevant consequence relation.

4. Supervenience and Relevant Entailment

An informational semantics of the first degree relevant logic was proposed and developed independently of each other by M. Dunn¹⁵ and E. Voishvillo¹⁶. It is based on a machinery of generalized state descriptions. 'Generalized' when applied to state descriptions means exactly the same as the result of rejection conditions (1) and (2) for object descriptions but now with respect to statements. We start with DEF.4, (1) and (2) omitted. It is quote

14 Nuel D. Belnap, "A useful four-valued logic," in *Modern uses of multiple-valued logic*, ed. J. Michael Dunn and George Epstein (Dordrecht: Springer, 1977), 5-37.

J. Michael Dunn, "Intuitive semantics for first-degree entailments and 'coupled trees,'" *Philosophical Studies* 29, no. 3 (Mar., 1976): 149-168.

15 J. Michael Dunn, "An intuitive semantics for first degree relevant implications," *Journal of Symbolic Logic* 36, no. 2 (1971): 362-363.

16 The first mention of such semantics is contained in the paper of Voishvillo published in 1976, in Russian. In the final form, information semantics is presented in E. K. Voishvillo, "Semantics of Generalized State Descriptions," in *Logic, methodology and philosophy of science VI*, vol. 104 of *Proceedings of the Sixth International Congress of Logic, Methodology and Philosophy of Science*, ed. L. Jonathan Cohen, et al. (Amsterdam: Elsevier, 1982), 315-323.

natural to formulate truth values assignment for formulas not in terms of corresponding four-valued function, but rather via relations connecting formula and generalized state description: $\mathbf{TA}/\alpha - A$ is true in α , and $\mathbf{FA}/\alpha - A$ is false in α .

DEF 7.

$$\mathbf{T}p/\alpha \Leftrightarrow p \in \alpha; \quad \mathbf{F}p/\alpha \Leftrightarrow \neg p \in \alpha;$$

$$\mathbf{T}\neg B/\alpha \Leftrightarrow \mathbf{F}B/\alpha; \quad \mathbf{F}\neg B/\alpha \Leftrightarrow \mathbf{T}B/\alpha ;$$

$$\mathbf{T}(B \wedge C) / \alpha \Leftrightarrow \mathbf{T}B/\alpha \text{ and } \mathbf{T}C/\alpha;$$

$$\mathbf{F}(B \wedge C) / \alpha \Leftrightarrow \mathbf{F}B/\alpha \text{ or } \mathbf{F}C/\alpha.$$

The definition of consequence relation coincides with a classical one.

DEF 8.

$$A \Vdash_{\text{FDE}} B \Leftrightarrow \forall \alpha (\mathbf{T}A/\alpha \Rightarrow \mathbf{T}B/\alpha)$$

Now DEF.6 may be rewritten in terms of first-degree entailment as follows:

DEF 9.

$$A \Vdash_{\text{SE}} B \Leftrightarrow \forall \alpha \forall \beta (((\mathbf{T}A/\alpha \text{ and } \mathbf{T}A/\beta) \text{ or } (\mathbf{F}A/\alpha \text{ and } \mathbf{F}A/\beta)) \Rightarrow ((\mathbf{T}B/\alpha \text{ and } \mathbf{T}B/\beta) \text{ or } (\mathbf{F}B/\alpha \text{ and } \mathbf{F}B/\beta)))$$

What we need now to establish correspondence between two semantical considerations, are straightforward **Principles of Propositional Reduction**:

$$\forall A((\mathbf{t} \in |A(k)|_d \text{ and } d(k)=\alpha) \Leftrightarrow \mathbf{T}A/\alpha)$$

$$\forall A((\mathbf{f} \in |A(k)|_d \text{ and } d(k)=\alpha) \Leftrightarrow \mathbf{F}A/\alpha).$$

These principles validate the
Correspondence Proposition: $A \models_{SE} B \Leftrightarrow A \models_S B$

Back to DEF.9, now we are in a position to present it as a conjunction of two conditions.

DEF 9*.

$$A \models_{SE} B \Leftrightarrow \forall \alpha \forall \beta ((\mathbf{T}A/\alpha \text{ and } \mathbf{T}A/\beta) \Rightarrow ((\mathbf{T}B/\alpha \text{ and } \mathbf{T}B/\beta) \text{ or } (\mathbf{F}B/\alpha \text{ and } \mathbf{F}B/\beta)) \text{ and } \forall \alpha \forall \beta (\mathbf{F}A/\alpha \text{ and } \mathbf{F}A/\beta) \Rightarrow ((\mathbf{T}B/\alpha \text{ and } \mathbf{T}B/\beta) \text{ or } (\mathbf{F}B/\alpha \text{ and } \mathbf{F}B/\beta)).$$

If considered independently, these two conditions conjunct in definiens may be interpreted as Positive Supervenience (PS) and Negative Supervenience (NS) correspondingly. To the best of my knowledge, they have never been differentiated before. However, it may be sometimes useful to make more sharp distinction between two situations: (PS) if any two things equally possess A -properties, then they are indiscernible with respect to B ; and (NS) if any two things equally do not possess A -properties, then they are indiscernible with respect to B . For a moment let me consider them separately.

DEF 10.

$$A \models_{PSE} B \Leftrightarrow \forall \alpha \forall \beta ((\mathbf{T}A/\alpha \text{ and } \mathbf{T}A/\beta) \Rightarrow ((\mathbf{T}B/\alpha \text{ and } \mathbf{T}B/\beta) \text{ or } (\mathbf{F}B/\alpha \text{ and } \mathbf{F}B/\beta))$$

It can be easily shown that $A \models_{PSE} B \leftrightarrow A \models_{FDE} B$ or $A \models_{FDE} \neg B$, reducing positive supervenience to relevant entailment relation.

DEF 11.

$A \vDash_{\text{NSE}} B \Leftrightarrow \forall \alpha \forall \beta ((\mathbf{FA}/\alpha \text{ and } \mathbf{FA}/\beta) \Rightarrow ((\mathbf{TB}/\alpha \text{ and } \mathbf{TB}/\beta) \text{ or } (\mathbf{FB}/\alpha \text{ and } \mathbf{FB}/\beta)))$

In its turn, for negative supervenience holds the following equivalence:

$A \vDash_{\text{NSE}} B \Leftrightarrow \neg A \vDash_{\text{FDE}} B \text{ or } \neg A \vDash_{\text{FDE}} \neg B.$

Thus, we have at our disposal everything we need to combine all the above into a final definition.

DEF 9.**

$A \vDash_{\text{SE}} B \Leftrightarrow A \vDash_{\text{FDE}} B \text{ or } A \vDash_{\text{FDE}} \neg B \text{ or } \neg A \vDash_{\text{FDE}} B \text{ or } \neg A \vDash_{\text{FDE}} \neg B.$

Some interesting properties of supervenience entailment relation can be clarified and illustrated by considering disjuncts in the definies. The first disjunct validates such characteristics of conjunction and disjunction as commutativity, associativity, distributivity and idempotence. Due to the second disjunct DeMorgan laws are valid as well as odd entailments of the form $A \vDash_{\text{SE}} \neg A$ and $\neg A \vDash_{\text{SE}} A$. Finally, last two disjuncts serve as a safeguard against invalid principles of Conjunction Introduction and Disjunction Elimination discussed above.

5. Conclusion

The proposed interpretation of supervenience as a specific (relevant) entailment relation though presumptive, demonstrates that certain simplified version of supervenience may be formally explicated via consequence relation, and, what is more important, in so doing one does not need to fall back on such essences as possible worlds. Instead I exploited an idea of vague objects and their descriptions, which turns us back to oldy but goldy state (and object) description device.

It is well known that possible worlds semantics are often criticized for appeal to formal concepts lacking informal (first of all, philosophical) interpretation. To name but a few, in *The Stanford Encyclopedia of Philosophy*, entry *Possible Worlds* by Menzel¹⁷ one can find the following passage:

Unfortunately, the semantics leaves the most interesting – and difficult – philosophical questions largely unanswered.

Two arise with particular force:

QW What, exactly, is a possible world?

And, given QW:

QE What is it for something to exist in a possible world?

From my point of view, corresponding questions resulting from the previous by replacing the expression ‘possible world’ with word-combination ‘vague object’ are easier to answer, because appropriate answers step from scientific research practice patterns.

Besides, bringing in philosophical discourse logical concepts like possible world or accessibility relation rises issues of what are the corresponding conditions on accessibility relation or what modal principles they validate and so forth. In a sense,

¹⁷ Christopher Menzel, “Possible Worlds,” in *The Stanford Encyclopedia of Philosophy*, Stanford University, 1997-, article published October 18, 2013; last modified February 8, 2016, <http://plato.stanford.edu/archives/fall2016/entries/possible-worlds/>.

such logical concepts purified from corresponding logical theory become opaque.

Definitely, current version of entailment interpretation of supervenience is only a touchstone of future work and should be considered as a request for further discussion.

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